CHAOS IN GDP GROWTH RATE OF G20 COUNTRIES

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Abstract. *This paper analyse the chaos in GDP growth rate of G20 countries specifically as these countries holds specific interest in the world's economic stability. Observation of deterministic chaos in the GDP growth rate helps to predict the nature of economy of a country for coming 5 to 10 years. We considered the data of real GDP growth of these countries from World Bank sited during the years 1961 to 2018. We use techniques such as time series plot, Phase portraits, Hurst exponent determination, Embedding dimension, Detrended fluctuation Analysis (DFA), Fractal dimension, and Lyapunov exponent for the analysis of the data. We have also conducted 0-1 test for deterministic chaos. Our study reveals a chaotic nature for the economic growth rate for most of the countries.*

Keywords: Hurst exponent, Chaos, Lyapunov exponent, Fractal dimension, DFA Analysis, GDP

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1. INTRODUCTION

The growth or decline in Gross Domestic Product (GDP) rate of a country has always been treated as a topic of study by many researchers. GDP is considered to be a very powerful measure to gauge the economic health of any country, since it reects the sum total of the production of the country [1, 2, 3]. The increase in GDP growth rate of a country depends on many factors such as industrialization and advances in technology, land and underground resources, capital formation, human resources etc. [4]. Many researchers investigate the impact of liberalization on the growth rate of GDP of various developing countries [5, 6]. Their studies reveal that for most of the developing countries, liberalization has a positive e_ect on domestic economic growth. But also found that growth itself has a negative impact on trade balance for a majority of countries.

The use of chaos theory in the analysis of economics problems has become an important area of research during the last few years [7, 8]. Applications of chaos theory in economics include topics such as explaining business cycles and forecasting movements in stock markets. Most of the studies involve the analysis of time series using nonlinear techniques [9, 10]. The application of the Lorenz attractor to model economic forecasting was studied by Michaela Simionescu in 2018 [11] and found that when specific values are given to the parameters in the Lorenz system, there is a tendency of increase in world economic growth on the horizon 2015-2019. Radko Kriz studied the presence of chaos in the GDP growth rate time series of seven European countries and established the existence of chaos in their GDP growth rate [7].

GDP growth rate is generally considered as a powerful indicator by most of the governments and economic decision-makers for planning and policy formulation. The GDP concept was first introduced by Simon Kuznets in a US Congress report in 1934 [12]. An unsteady growth of GDP per capita of a country may lead to increase of poverty, reduction in social health and education, increase in crime and eventually reduces the economic growth. The geometric annual growth rate in GDP over a period of time between first and last year is known as ***rate of economic growth***. This rate ignores any fluctuations in the trend in the average level of GDP over the period. Various surveys report that liberalization alone has positive impact on GDP growth in a short span of time but combined with income terms of trade yields a negative overall impact on trade balance to GDP percentages [7, 13] in the case of developing countries

Even though various formal governmental organizations and international groups that have been formed to discuss the global financial stability, the Group of Twenty (G20), founded in 1999, allows a forum to discuss global economic growth, as well as regulations concerning financial markets. After the acceptance of Paris Climate Agreement in 2015, issues of global significance are also included in their objectives. Almost two-thirds of the world's total population is represented by G20 countries and they indicate 85 percent of the total global gross domestic product and more than 75 percent of the global trade. That is why we select GDP growth rate of G20 countries for our study. In this study, we try to investigate the relationship between liberalization and GDP growth rates of G20 countries in a long time span and hence try to predict the global economic stability.

In this paper, we investigate the presence of chaos in the GDP growth rate of the G20 countries during the period 1961–2018 using various analysis techniques. We started with time series analysis, then constructed phase portraits and calculated fractal dimension, time delay, embedding dimension and estimated Hurst exponent. These are needed for the determination of Lyapunov exponent. We have also carried out the Detrended Fluctuation Analysis (DFA). Lyapunov exponent is considered as an important parameter for the identification of chaotic nature of a dynamical system. Hurst exponent is viewed as a numerical estimate of predictability of time series. We have conducted 0–1 test for deterministic chaos with time series of the GDP growth data as input. The general trend we found is an inverse dependence of GDP rate on the Lyapunov exponent. A detailed account of the methods of analysis and results are given in the following sections. In the concluding section, we point out an economy with a positive Lyapunov exponent as chaotic, whereas that with a negative value is subjected to control. The inverse relation obtained between growth rate and Lyapunov exponent is rather an unexpected one. Since the real GDP accounts for changes in market value, it narrows the difference between output figures from year to year. The existence of deterministic chaos in a GDP growth rate allows us to predict the future behavior of the economic state of a country based on the previous policies based on liberalization, global economy and other factors adopted by the government, to some extent.

2. METHODS OF ANALYSIS

For analyzing the behaviour of the GDP growth rate of G20 countries, we have employed various nonlinear methods. These techniques help us to investigate the chaotic nature of GDP growth of these countries. The techniques used for nonlinear analysis are described below.

2.1 Time series analysis

A set of observations or a sequence of data recorded over regular intervals of time is known as time series. Time series data can be used for analyzing the trend, seasonal fluctuations and residual variations for various types of data. Since time is a physical concept, the parameters and other characteristics in the mathematical models for time series can have real world interpretations. Depending on the field of application, time series analysis has many different objectives. Here we use the real GDP annual growth rate data taken from the World Bank Site [14] for creating time series of G20 countries. We considered the real GDP annual growth rate during the period 1961 – 2018 for all countries except Germany, Russia and Saudi Arabia. For Germany we considered the data during 1971 – 2018, for Russia it is during 1990 – 2018 and for Saudi Arabia we took data during 1969 – 2018 for our analysis since only data during these time period are available. The consolidated time series plot of all the G20 countries is shown in (Figure 1).

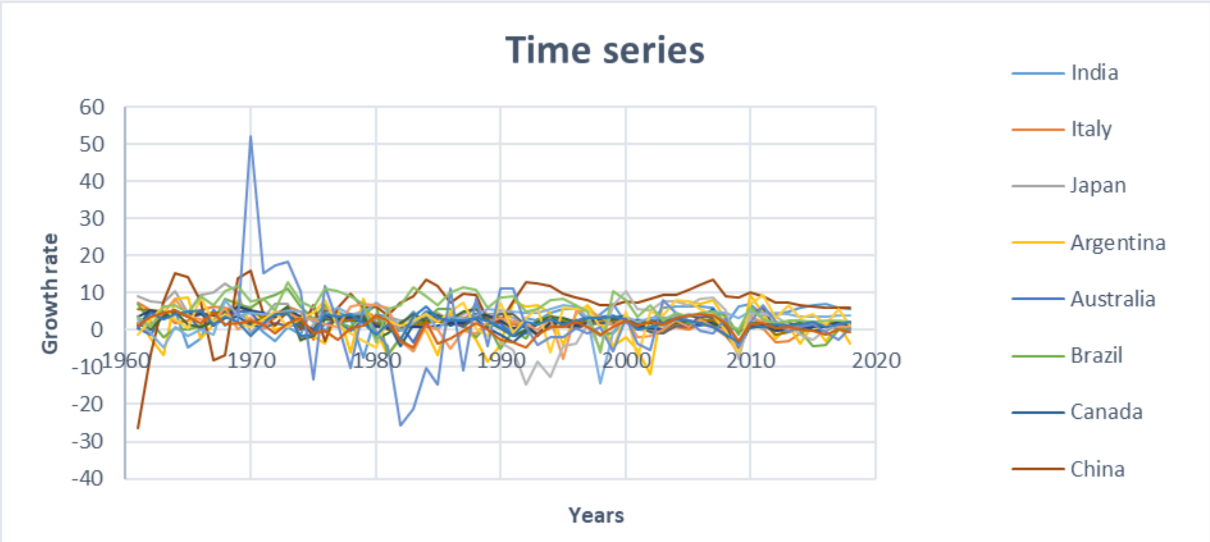


Fig 1: Timeseries of G20 countries

2.2 Reconstruction of Phase Space

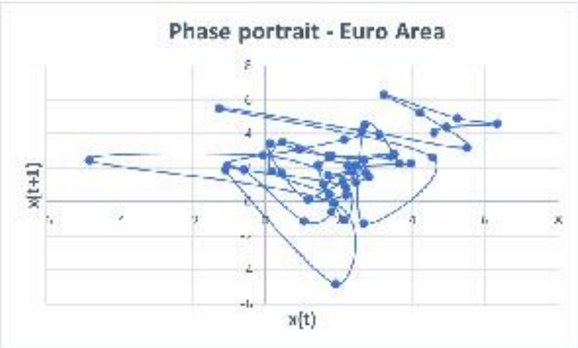
To understand the behavior of G-20 countries from the time series data, it is very important to reconstruct the phase space. Reconstruction of phase space leads to the representation of complete dynamics using a single time series. For a proper reconstruction, it is very essential to know the optimum time delay (τ) and the embedding dimension (m).

For a dynamical system, whole information about the variable is present in the univariate time series. Each point of phase space represents a state of the system, while a trajectory in the phase space represents the time evolution of the system, according to different initial conditions [10, 12, 15, 16, 17].

A phase space can be created from a one-dimensional time series using Takens time delay embedding theorem [12]. For a chaotic system with scalar time series T_t =

$\{N_1, N_2, N_3, \ldots\}$ the reconstruction is possible with the phase space vectors $X(t)$ expressed as : $X(t) = [x(t), x(t + \tau), \ldots, x(t + (m - 1)\tau)]$ where $t = 1, 2, \ldots, M$; $M = N - (m - 1)\tau$ where τ is the time delay, m embedding dimension of Phase space reconstruction, and M is the number of phase points of reconstructed phase space. Since we are estimating τ for nonlinear time series we use average mutual information (AMI) in the present work. The first local minimum of AMI is chosen as the optimum time delay.

As an example, the Phase portrait constructed for Euro Union and India are given in (Figures 2a and 2b).



Fig, 2(a) Phase portrait of Euro Union

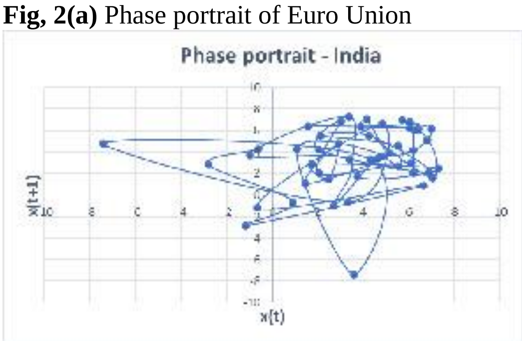


Fig 2b. Phase Portrait of India

2.3 Embedding Dimension

In this investigation we adopted a practical method to determine the minimum embedding dimension from a scalar time series using Takens theorem [18]. Four basic methods that are used to find the minimum embedding dimension: computing some invariant on the attractor, singular value decomposition, the method of false neighbors [17], and Cao's method [19]. We use Cao's method to determine the embedding dimension. Using this method, we can distinguish deterministic signals from stochastic signals. It does not depend on how many data points are available. There are two quantities $E1$ and $E2$, where $E1$ gives the embedding dimension required for the reconstruction and $E2$ helps to distinguish deterministic signals from stochastic signals. For random signals, the value of $E2 = 1$, for every value of m.

2.4 Pearson Correlation Coefficient

The Pearson correlation coefficient is a measure of linear relationship between two data variables X and Y having the values in the range between -1 and $+1$. When $r = 0$, it indicates that there is no relationship between two variables such that the change in magnitude of one variable is independent of that in the other variable. When $r > 0$, it is said to be positively correlated, and there is a direct relationship between the two variables. As the magnitude of the one variable increases the other also increases. Similarly, if the magnitude in one variable increases and that in the other decreases, the relationship between two variables is said to be negatively correlated and this is for $r < 0$. The mathematical formula for calculating the Pearson correlation coefficient [20, 18, 21, 22] is

$$r = \frac{\sum\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sum\left(X_i - \overline{X}\right)^2 \sum\left(Y_i - \overline{Y}\right)^2} \tag{1}$$

Ranges from -1 (a perfect negative correlation) to $+1$ (a perfect positive correlation). When $r = 0$ no correlation exists.

2.5 HURST EXPONENT (HE)

The Hurst exponent provides a measure of autocorrelation [23, 24, 25]. The value of the Hurst exponent (H) ranges between 0 and 1. For random time series data or white noise the value of H is 0.5, antipersistant or no correlation ($H < 0.5$) and persistent or correlation ($H > 0.5$). The relation between Hurst exponent H and fractal dimension D_f is given by, $D_f = 2-H$, which measures the intensity of long-range dependence in a time series. In this method, a time series of length of L is divided into d subseries of length n. For each subseries $m = 1, \ldots.\ d$, find the mean (E_m) and standard deviation (S_m); Then after normalizing the data, a cumulative time series is created as

$$Y_{i;m} = \sum_{j=1}^{i} x_{j,m} \qquad i = 1,2,3,\ldots\ldots, m \tag{2}$$

and also the range $R_m = max\,(Y_{l,m} \ldots, Y_{n,m}) - min\,(Y_{l,m} \ldots, Y_{n,m})$. Then rescale the range R_m/S_m. The mean value of the rescaled range for all subseries of length n can be calculated as ;

$$\left(\frac{R}{S}\right)_n = \frac{1}{d}\sum_{m=1}^{d}\frac{R_m}{S_m} \tag{3}$$

The ratio (R/S) asymptotically follow the relation [26]

$$\left(\frac{R}{S}\right)_n \approx cn^H \tag{4}$$

It follows that,

$$\log\left(\frac{R}{S}\right)_n = H\log n + \log c \qquad (5)$$

Or

$$H = \frac{\log\left(\frac{R}{S}\right)_n - \log c}{\log n} \qquad (6)$$

2.6 Fractals and Dimension (FD)

Infinitely complex patterns that have self-similar structures across different scales are called fractals. In mathematics, fractals are considered as a class of complex geometric shapes that have a fractional dimension called *fractal dimension* (FD). In 1918, this concept was first introduced by the mathematician Felix Hausdorff [27]. The index for characterizing fractal patterns or sets and can be calculated by quantifying their complexity as a ratio of the change in detail to the change in scale. It gives a measure of the complexity of the phase portrait of the time series which indicates how many independent variables are needed to simulate the time series. The number of independent variables required is calculated from the fractal dimension by rounding it up to the next integer.

2.7 Detrended Fluctuation Analysis (DFA)

The complex temporal structure of ongoing oscillations is generally scale free and characterized by long-range temporal correlations. Detrended fluctuation analysis (DFA) is very effective in measuring the persistency (or antipersistency) of data series with non-stationarities [11]. Hence it can be analyzed using DFA which lead to differences in the scale-free amplitude modulation of oscillations. For a given time series data $B(i)$, the average value, B_{avg}

$$B_{avg} = \frac{1}{N}\sum_{i=1}^{N} B(i) \qquad (7)$$

The cumulative sum of $B(i)$ obtained by considering time series of the total length N is integrated

$$y(k) = \sum_{i=1}^{k} (B(i) - B_{avg}) \qquad (8)$$

This integration steps maps the original time series to a self-similar process. Fluctuation $F(n)$ employed in the DFA obtained by dividing the integrated time series into small equal parts of width n and then the time series $y(k)$ is detrended by subtracting the local trend $y_n(k)$ in each n. For a given size n, the characteristic size of fluctuation for this integrated and detrended time series is calculated by

$$F(n) = \sqrt{\frac{1}{N}\sum_{k=1}^{N}\left[y(k) - y_n(k)\right]^2} \qquad (9)$$

where n is the window size and $y(k)$ is the linear function which approximated by the least squares method for each window. The fluctuation $F(n)$ is proportional to the power of n, $\Longrightarrow F(n) \propto n^{\alpha}$. where α is known as the scaling exponent and is utilized to characterize the fluctuation as shown in (Table 1).

We use DFA method to analyze the long-range correlation and scaling properties of annual GDP growth rate. The slope of the line relating log $F(n)$ versus log n graph determines the scaling exponent (self-similarity factor).

Table 1: Classification of α value

α Value	**Classification**
$\alpha < 0.5$	Anticorrelated
$\alpha = 0.5$	Uncorrelated, White noise
$1 < \alpha < 0.5$	Correlated
$\alpha = 1$	Pink noise
$1 < \alpha < 1.5$	Random walk
$\alpha = 1.5$	Brownian noise

2.8 Lyapunov Exponent

In a dynamical system the Lyapunov exponent characterizes the rate of separation of infinitesimally close trajectories. The instability in chaotic systems is usually characterized by the spectrum of Lyapunov Exponents. Lyapunov exponents (λ) are considered as a reliable measurement to predict the nature of a dynamical system since they are invariant under smooth transformations and hence are independent of the measurement function or the embedding procedure. If $F^t(x_0 + s)$ and $F^t(x_0)$ are two close trajectories, then

$$F^t(x_0 + \varepsilon) - F^t(x_0) \approx \varepsilon e^{\lambda t} \qquad (10)$$

Here λ is called Lyapunov exponent. When $\lambda > 0$, small distances grow indefinetly over time, which is an indicator of the onset of chaos. Also when $\lambda < 0$, the system settles down into a periodic trajectory eventually.

In this study, we use the algorithm of Rosenstein [26] to evaluate Lyapunov exponent by considering the reconstructed phase space. In the reconstructed phase space, the nearest neighbour (X_i) of each point on the trajectory is located by looking for the point which has minimum distance from the reference point (X_j)

$$d_j(0) = \min_i \left| X_i - X_j \right| \qquad (11)$$

where $d_j(0)$ is the initial distances from i^{th} point to its neighbour. Similarly nearest neighbour is found for other points also by repeating this process. The distance between reference point and nearest point after i iterations may be denoted by $d_j(i)$. Then

$$y(i)=\frac{<\ln(d_j(i))}{i\nabla t} \quad (12)$$

where Δt is the sampling period of time series data. The largest Lyapunov exponent is the slope of the log < *divergence* > vs *time*(*s*) graph in increasing part

2.9 The 0-1 Test for Chaos

Another test to study the onset of chaos was developed by Gottwald and Melbourne [28] to distinguish between the periodic (regular or quasi-periodic) and chaotic behavior from a discrete and continuous dynamical system. This test measures directly from the one-dimensional scalar time series and does not require any reconstruction of phase space which is necessary for calculating Lyapunov exponent. The output of this test gives a scalar value in between 0 and 1 where the value of '0' indicates a periodic dynamics and the value of '1' indicates a chaotic dynamics. The algorithm of 0-1 test is as follows :

Step 1: From a given one dimensional scalar time series $\varphi(n)$ for $n = 1, 2, \dots, N$, a new two-dimensional coordinate system is obtained by $P(n)$ and $q(n)$ as

$$P(n)=\sum_{i=1}^{n}\phi(i)\cos(ic);\quad q(n)=\phi(i)\sin(ic) \quad (13)$$

where c is a random number which is fixed in the range

$$c\in\left(\frac{\pi}{5},\frac{4\pi}{5}\right)$$

Step 2: Define the Mean square displacement $M(n)$ as,

$$M(n)=\lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\Big(\big(p(i+N)-p(i)\big)^2+\big(q(i+n)-q(i)\big)^2\Big) \quad (14)$$

$$\text{for } n\in\left(1,\frac{N}{10}\right)$$

Step 3: Define the modified Mean square displacement $D(n)$ as

$$D(n)=M(n)=\left(\lim_{N\to\infty}\sum_{i=1}^{N}\phi(i)\right)^2\frac{1-\cos nc}{1-\cos c} \quad (15)$$

where α =1,2,3,….,N/10 and δ = D(1), D(2),…D(N/10)

Step 4 Estimate the correlation coefficient of the linear growth rate as

$$R=\frac{\operatorname{cov}(\alpha,\delta)}{\sqrt{\operatorname{var}(\alpha)\operatorname{var}(\delta)}}\in(-1,1) \quad (16)$$

where $\alpha = 1, 2, \dots, (N/10)$ and $\delta = D(1), D(2), \dots, D(N/10)$. Then K = median(R)

Table 2. Consolidated result of nonlinear Analysis

Sl. No.	Country	Total years	Correlation coeft.	Hurst Exponet	DFA	Fractal Dimension	0-1 Test
1	Argetina	58	0.114	0.47	0.82	1.53	0.99
2	Australia	58	0.168	0.58	0.86	1.42	0.99
3	Brazil	58	0.475	0.68	1.11	1.33	0.99
4	Canada	58	0.279	0.67	0.91	1.33	0.99
5	China	58	0.507	0.55	0.95	1.45	0.97
6	Euro Area	58	0.562	0.57	1.11	1.43	0.98
7	France	58	0.587	0.34	1.18	1.66	0.97
8	Germany	58	0.142	0.69	1.04	1.31	0.99
9	India	58	0.174	0.32	0.81	1.68	0.99
10	Indonesia	58	0.344	0.63	1.13	1.37	0.98
11	Italy	58	0.500	0,29	0.97	1.71	0.99
12	Japan	58	0.578	0.63	0.85	1.37	0.99
13	Korea	58	0.335	0.35	0.80	1.65	0.99
14	Mexico	58	0.240	0.59	0.99	1.41	0.99
15	Russia	28	0.653	0.77	1.24	1.23	089
16	Saudi Arabia	49	0.298	0.43	1.01	1.58	0.99
17	South Africa	58	0.512	0.62	1.05	1.38	0.66
18	Turkey	58	-0.009	0.47	0.89	1.53	0.97
19	UK	58	0.347	0.45	1.01	1.55	0.97
20	US	58	0.288	0.63	0.94	1.37	0.99

3.RESULT ANALYSIS OF GDP TIME SERIES DATA OF G20 COUNTRIES

The annual real GDP growth rate data obtained from the World Bank site for G20 countries are analysed using the parameters described above, and the results are consolidated in (Table 2). There are deterministic systems whose time evolution has a very strong dependence on initial conditions and they are familiarized as chaotic systems and can predict a deterministic chaos [29]. This is because, these models will always produce the same output from a given starting condition or initial state. But a random or stochastic process depends on a collection of random variables, representing the evolution of some system for random values over time.

The phase portraits and fractal dimension indicate a chaotic nature of GDP growth rate for all the G20 countries. The 0–1 test confirms this result. But the Lyapunov analysis and Hurst exponent values indicate that some of the countries like Australia, India, Russia, UK, USA and Euro Union have a deterministic nature while the some other countries shows stochastic nature.

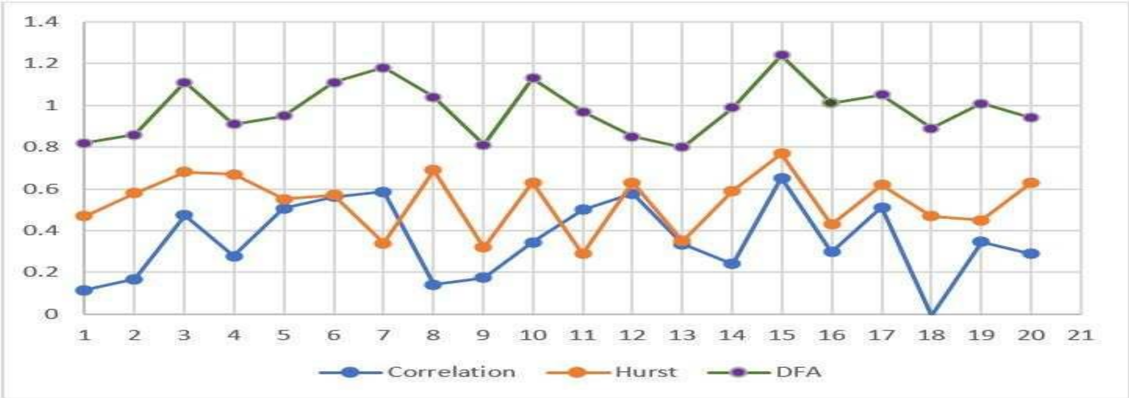


Fig.3: Comparison of Correlation coefficient, Hurst Exponent and DFA values

A plot with correlation, Hurst exponent and DFA values on Y axis and the G20 countries from 1 to 20 on X axis is given in (Figure 3) for acomparison. The behaviour of these values for certain countries shows a trend towards stable economy, while for certain other countries shows a random behavior.

Generally we need to have a large number data for analysing chaos in GDP in a deep sense. But the annual GDP data available is only from 1961 to 2018 with only 58 data points. Eventhough the analysis of such short time series may be questionable, we donot have other choice than to analyse the chaotic behaviour of GDP growth rate of time series. Hence the results we present are only approximations. We try to estimate the trend only from which future trend also can be predicted provided the initial conditions remains same. The plot of Lyapunov exponent of plotted for all the G20 countries based on the 58 points is shown in Figure 4. We have also compared the Lyapunov exponent slope and rate of economic growth slope for these countries from 1961 to 2018. In this case we could find an inverse relation between Lyapunov exponent and economic growth rate for most of the countries during this time span. Few of the countries show a random nature. When consider short time span of two or three years, there are uctuations in these values from the average value, still with a reverse relation. To get a closer inference, we consider the slope of Lyapunov exponent and GDP growth for the time span 2010- 2018. A detailed analysis on the slope of the economic growth rate and the average of Lyapunov exponent slope is given in the next section.

Fig. 4: Lyapunov exponents of G20 countries comsolidat

3.1 SLOPE ESTIMATION FROM 2010 TO 2018

A histogram showing the consolidated contribution of slope in percentage of all G20 countries is shown in (Figure 5).

Twelve out of the twenty countries shows an inverse relation between economic growth rate and Lyapunov exponent and this result is tabulated in (Table 3). The sign of Lyapunov exponent is very important in understanding the dynamics of the system, whether the system leads to a chaotic state or not.

Detailed analysis of the data shows that, out of the twelve countries, Australia, Euro Union, India, Italy, UK and USA shows a negative Lyapunov exponent value with a positive trend for the economic growth rate during 2010 to 2018.

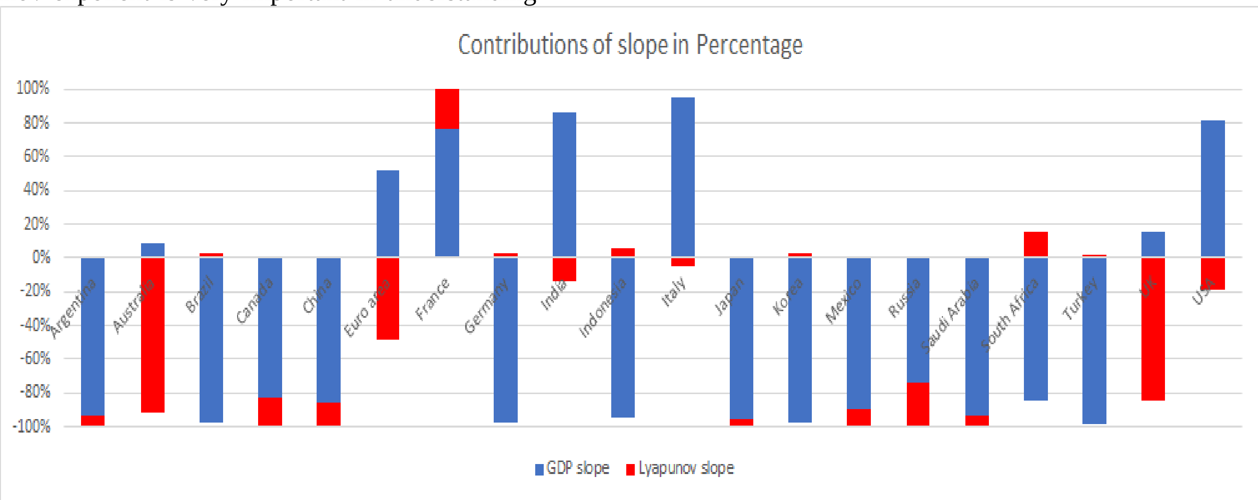


Fig. 5: Contributions of slope in percentage

This result gives a confrmation to our assumption that liberalization in economy, along with other factors, has a major role in controlling the growth of GDP in the long run. This result also indicate that the government policies also have a role in the GDP growth since usually one government continues for 4 to 5 years in most of these countries. Here the important result is that, among these six countries, five are developed countries and India is the only developing country. Even though there are fluctuationss in GDP growth rate for short span of time line (two or threeyears), the lower end of the uctuation shows a positive growth trend when we consider the economic growth rate of eight or ten years with a deterministic nature for its economy. India after liberalization shows a continuous growth on an average GDP growth rate especially from 1999, accepting the fact that in certain years there are variations from the average value.

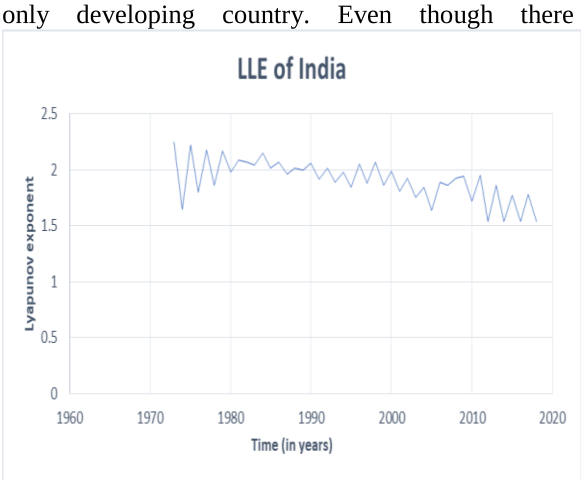


Fig 6. (a) Lyapunov exponent: India

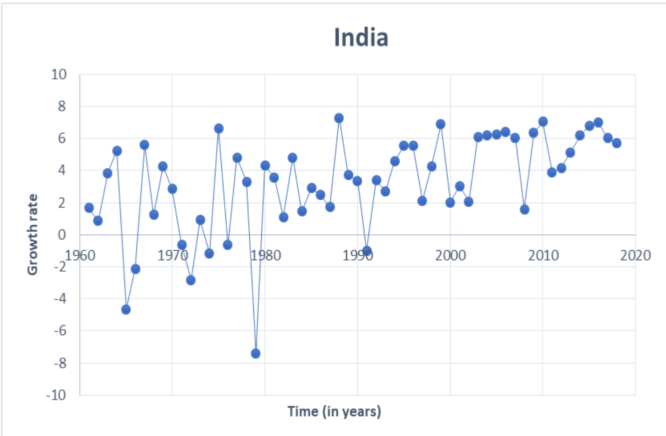


Fig. 6(b) Annual Real GDP growth rate of India from 1961 to 2018

Table 3. Inverse Relation between economic growth rate and Lyapunov exponent

Country	Economic growth rate	Lyapuniv Exponent
Australia	Positive	Negative
Euro area	Positive	Negative
India	Positive	Negative
Italy	Positive	Negative
UK	Positive	Negative
USA	Positive	Negative
Brazil	Negative	Positive
Germany	Negative	Positive
Indonesia	Negative	Positive
Korea	Negative	Positive
South Africa	Negative	Positive
Turkey	Negative	Positive

Euro Union has been considered as the second largest economic power after US. Also it is the only non state member in G20 group. Our analysis of Euro Union gives almost same magnitude for GDP growth rate and Lyapunov exponent with opposite signs (Figure 3 and figure 7). The time series analysis during 2010-2018 gives a negative Lyapunov exponent value with a positive economic growth rate as expected. This indicate that they are growing towards a stable economy, may be due to the importance they given to the constitutional law as a source for global governance guidance, the diversity of approaches they took and the environmental action they considered, respecting the G20 observations and decisions after each summit.

Fig, 7(a) Lyapunov exponent-Euro Union

Fig. 7(b) Annual Real GDP growth rate of Euro Union from 1961 to 2018

For the other six countries among the twelve - which include Brazil, Germany, Indonesia, Korea, South Africa and Turkey - the Lyapunov exponent give positive values and shows a decrease in GDP growth rate. Positive value of Lyapunov exponent is an indicator of the onset of chaos. Hence economy of these countries may lead to a chaotic state. The chaotic nature of their economy is confrmed using 0-1 test. Other eight countries shows negative values for both GDP growth rate and Lyapunov exponent except France. France shows a positive value for both GDP Growth rate and Lyapunov exponent. All these countries shows a random nature in their economy as seen from the most of the analysis results.

4. CONCLUSION

The study of economic growth theory itself is a complex process since it depends on many direct factors such as human resources, natural resources, technological advances and indirect factors such as efficiency of financial system, fiscal and budgetary policies, liberalization in economy and the efficiency of the government. Economic growth of a country is measured by the variation of annual GDP growth rate of that country but it is very difficult to determine the factors causing the variation. The analysis of chaos in GDP data for all the G20 countries is very di_cult to carry out because all these countries have independent government policies, climate conditions an economic policies. Stillwe are trying to and similarities in their economy because all these countries have some common objectives towards keeping global economic stability. We considered various nonlinear techniques such as time series, phase portraits, fractal dimension, correlation dimension, Lyapunov exponent, Hurst exponent and DFA and the analysis results are given in (Table 2) and (Figures 1, 2a, 2b, 3, 4 and 5). All the techniques shows a chaotic behaviour for most of the countries. But for few countries, the Hurst exponent, DFA, correlation function and Lyapunov exponent give a random or stochastic nature which is clear from (Figures 1, 2a and 2b). We have conducted 0-1 test for chaos using the time series of the GDP growth rate values. All the countries shows a chaotic nature. We considered the slope estimation of economic growth rate and Lyapunov exponent during 2010 to 2018 and the result is shown in figure 3. A close observation of the slope of their average GDP growth rate and slope of the average of Lyapunov exponent from 1961 to 2018 shows that these average values during a period of time (may be four or five years) depends on the previous value with the same period which showing a deterministic chaotic nature.

Summerizing, we observed that as the Lyapunov exponent increases from negative to positive value, the GDP growth rate decreases. It appears that although liberalization promotes growth in the short term, the growth rate can be sustained only by introducing some form of consultation among the state holders with the government. In other words, some degree of planning based on the nonlinear analysis may be helpful in achieving high economic goals and a sustained GDP growth rate. This is only an observation which has to be substantiated by a consideration of the economic history of the analysing country.

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6. REFERENCES

[1] Simon Kuznets. National income, 1929-1932. *National Bureau of Economic Research*, 49:1-12, 1934.

[2] H E Hurst. Long-term storage capacity of reservoirs. *Transactions of the American Society of Civil Engineers*, 116(1):770-808, 1951.

[3] Gazi Mainul Hassan and Shamim Shakur. *Nonlinear effects of remittances on per capita gdp growth in Bangladesh. Economies*, 5(3):1-11, 2017.

[4] Florin Teodor Boldeanu and Liliana Constantinescu. The main determinants affecting economic growth. *Bulletin of the Transilvania University of Brasov Series*, 8(57):329-338, 2015.

[5] Ashok Parikh and Corneliu Stirbu. *Relationship between trade liberalisation, economic growth and trade balance: An econometric investigation.* Hamburgisches Welt-Wirtschafts-Archiv (HWWA) Hamburg Institute of International Economics discussion paper, (282), 2004.

[6] Giovanni Andrea Cornia. The impact of liberalisation and globalisation on income inequality in developing and transitional economies. *Center for Economic Studies and ifo Institute (CESifo)*, Munich, 5(843), 2003.

[7] Radko Kriz and Radka Knezackova. The presence of chaos in the gdp growth rate time series. *Chaotic Modeling and Simulation*, 2:199-206,2014.

[8] Radko Kriz. Finding chaos in Finnish gdp. *International Journal of Automation and Computing*, 11(3):231-240, 2014.

[9] Karen Dynan and Louise Sheiner. Gdp as a measure of economic well being. *Hutchins Center on Fiscal and Monetary Policy, The Brookings Institution*, 43:1-52, 2008.

[10] Mathew B Kennel, Reggie Brown, Henry D, and I Abarbanel. *Determining embedding dimension for phase space reconstruction using a geometrical construction*. Physical Review A, 45(6):3403-3411, 1992.

[11] Jianhai Yue, Xiaojun Zhao, and Pengjian Shang. *Effect of trends on detrended uctuation analysis of precipitation series*. Mathematical Problems in Engineering, 2010.

[12] Floris Takens. Detecting strange attractors in turbulence. In Young LS.Rand D., editor, *Dynamical Systems and Turbulence, Warwick 1980- Lecture Notes in Mathematics*, vol 898, pages 366-381. Springer Nature, 1981

[13] Ashok Kotwal, Bharat Ramaswami, and Wilima Wadhwa. *Economic liberalization and Indian economic growth: What's the evidence?* Journal of Economic Literature, 49(4), 1152-1199, 2011

[14]World Bank.World bank open data https://data.worldbank.org.

[15] Syed Ahsin Ali Shah, Wajid Aziz, Maliksajjad Ahmed Nadeem, Majid Almaraashi, Seong O Shim, and Turki M Habeebullah. *A novel phase space reconstruction - (psr-) based predictive algorithm to forecast atmospheric particulate matter concentration*. Scientific Programming, 2019.

[16] Liangyue Cao. Practical method for determining the minimum embedding dimension of a scalar time series. Physica D, 110, 43-50, 1997.

[17] Andrew M Fraser and Harry L Swinney. *Independent coordinates for strange attractors from mutual information*. Physical Review A, 33(2), 1134-1140, 1986.

[18] Richard Taylor. *Interpretation of the correlation coefficient: A basic review*. Journal of Diagnostic Medical Sonography, 6(1):35-39, 1990.

[19] K Hema Divya and V Rama Devi. *A study on predictors of gdp: Early signals*. Procedia Economics and Finance, 11:375-382, 2014.

[20]Richard Hardstone, Simon-Shlomo Poil, Richard Hardstone, Simon- Shlomo Poil, Giuseppina Schiavone, Rick Jansen, Vadim V Nikulin, Huibert D Mansvelder, and Klaus Linkenkaer-Hansen. *Detrended fluctuation analysis : A scale-free view on neuronal oscillations*. Frontiers in Physiology, 3(450), 2012.

[21] Joseph Lee Rodgers and W Alan Nicewander. *Thirteen ways to look at the correlation coefficient*. The American Statistician, 42(1), 59-66, 1988.

[22] Patrick Schober, Christa Boer, and Lothar A Schwarte. *Correlation coefficients: Appropriate use and interpretation*. Anesthesia & Analgesia, 126(5):1763{1768, 2018.

[23] H E Hurst. *A suggested statistical model of some time series which occur in nature.* Nature, 180:494-495, 1957.

[24] Rafa Weron. *Estimating long-range dependence: finite sample properties and confidence intervals*. Physica A : Statistical Mechanics and its Applications, 312(1-2), 285-299, 2002.

[25] H E Hurst. *Problem of long term storage in reservoirs. International Association of Scientific Hydrology*. Bulletin, 1(3):13-27, 1956.
[26] Michael T Rosenstein, James J Collins, and Carlo J De Luca. *A practical method for calculating largest lyapunov exponents from small data sets.* Physica D: Nonlinear Phenomena, 65(1-2), 117-134, 1993.
[27] John E Hutchinson. *Fractals and self similarity*. Indiana University Mathematics Journal, 30(5), 713-747, 1981.
[28] Georg A Gottwald and Ian Melbourne. *A new test for chaos in deterministic systems*. Proceedings of The Royal Society A Mathematical Physicaland Engineering Sciences, 46 0(2042), 603-611, 2004.
[29] Mauro Cattani, Ibere Luiz Caldas1, Silvio Luiz de Souza, and Kelly Cristiane Iarosz1. *Deterministic chaos theory : Basic concepts*. Revista Brasileira de Ensino de Fisica, 39(1), 2017.